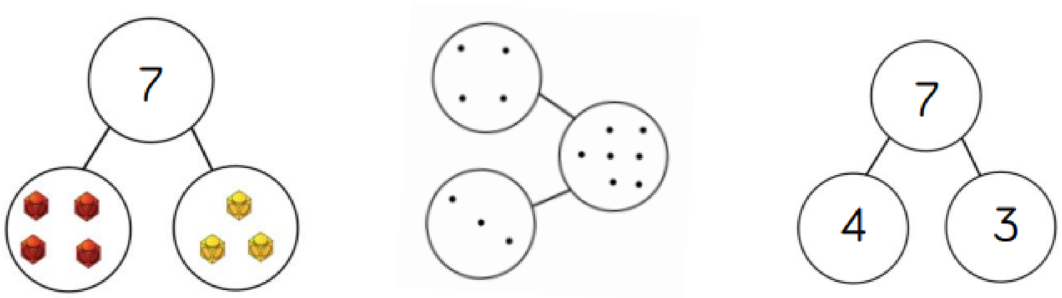
Parsloes Primary School

Maths Calculation Policy

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added.

It is a working document and will be revised and amended as necessary.

Part-Whole model

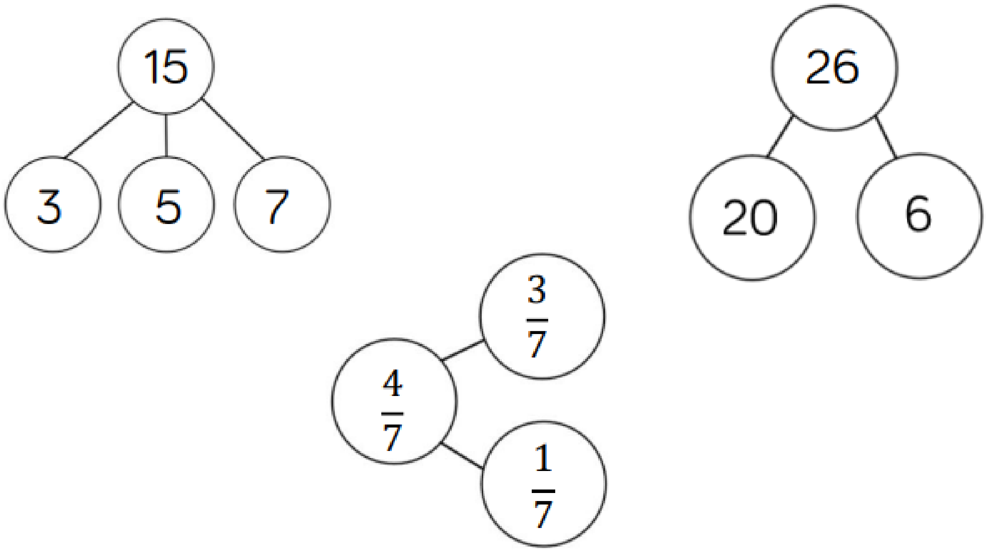


$7 = 4 + 3$

$7 = 3 + 4$

$7 - 3 = 4$

$7 - 4 = 3$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

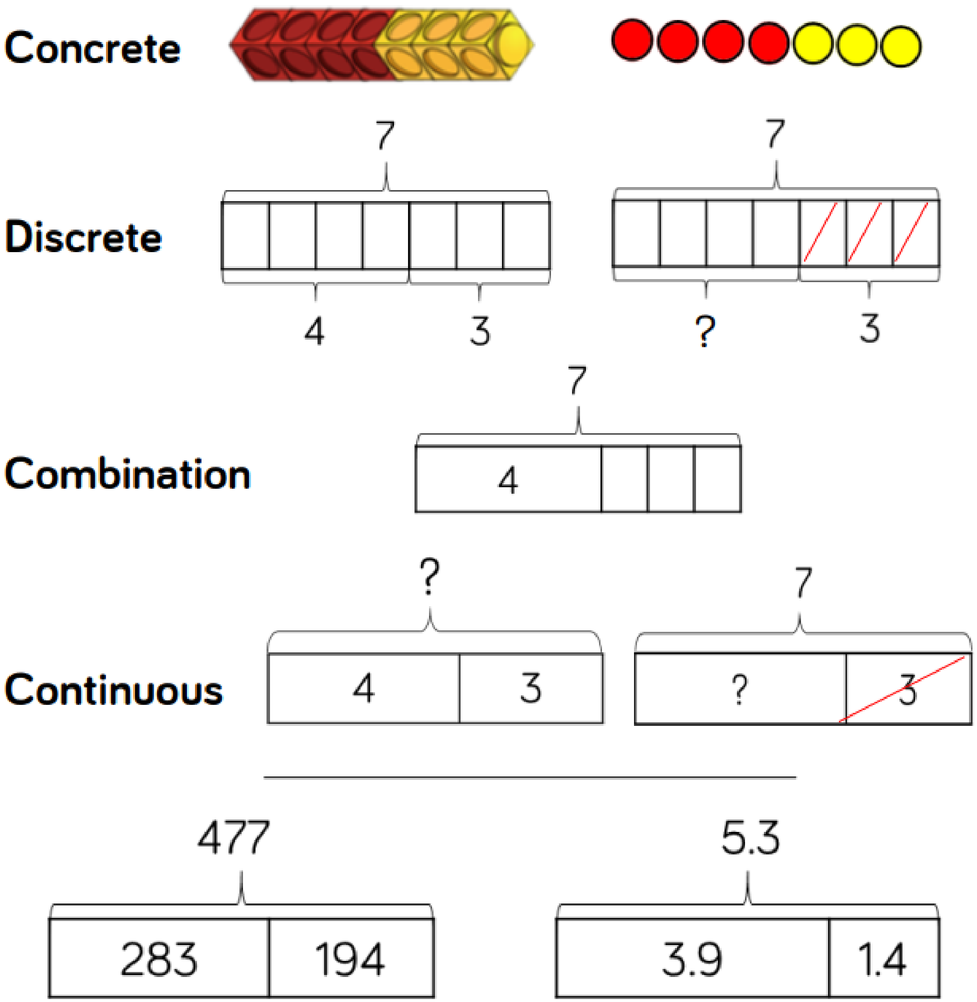
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

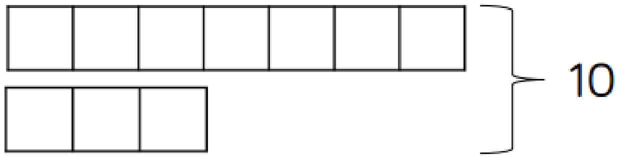
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

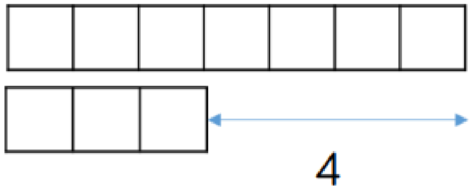
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar model (multiple)

Discrete

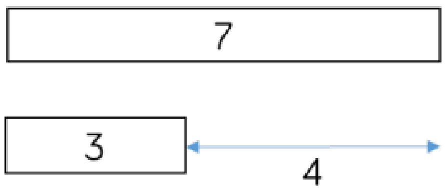


$7 + 3 = 10$

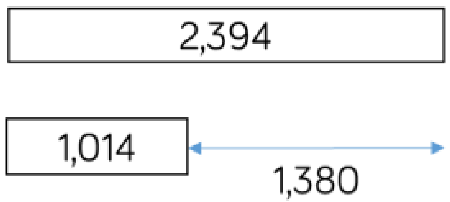


$7 - 3 = 4$

Continuous



$7 - 3 = 4$



$2{,}394 - 1{,}014 = 1{,}380$

Benefits

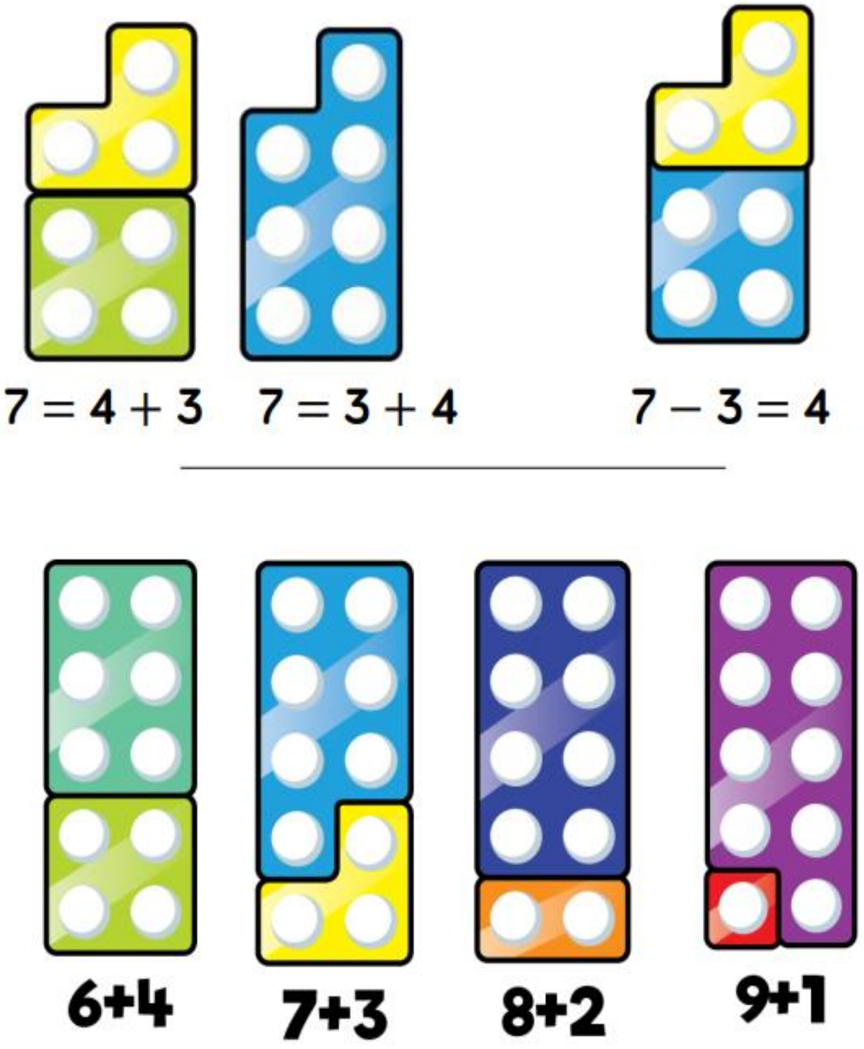
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Number Shapes



Benefits

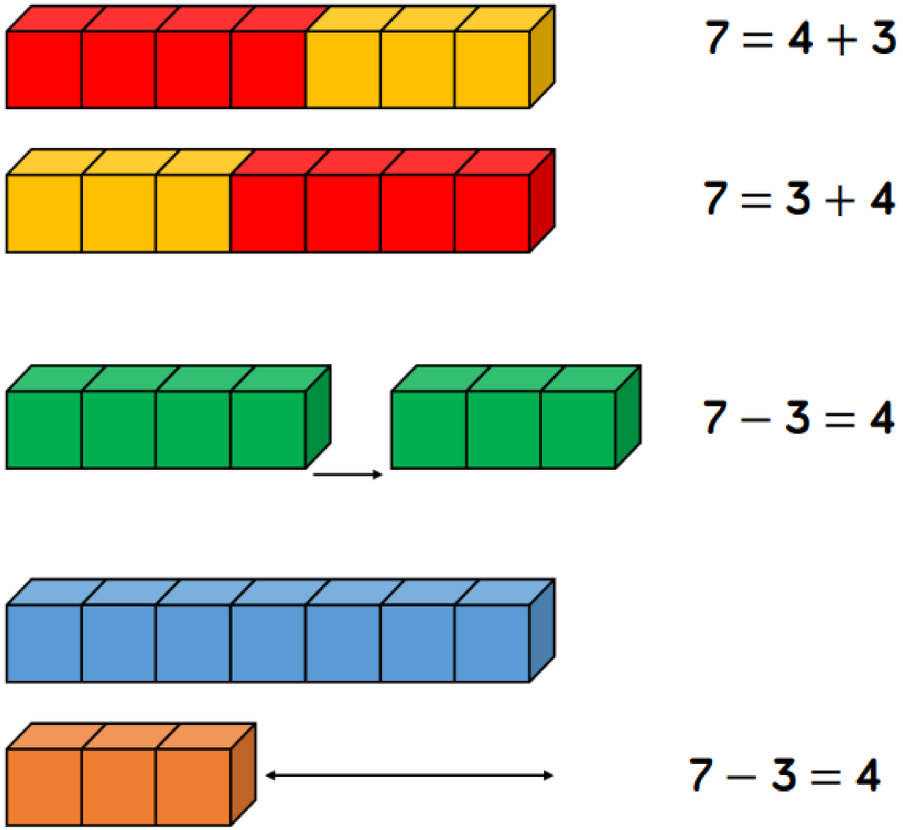
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

Cubes



Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

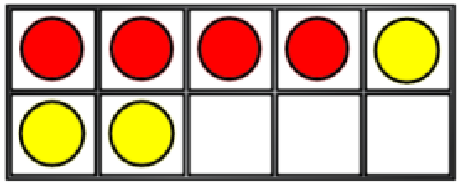
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

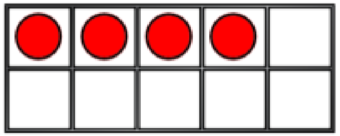
Ten Frames (within 10)

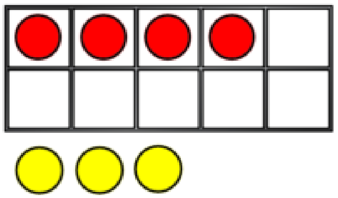


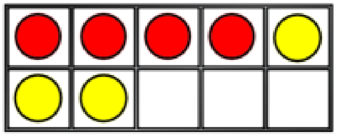
$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

4 is a part.
3 is a part.
7 is the whole.

First | Then | Now

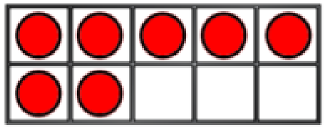


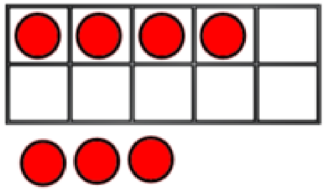


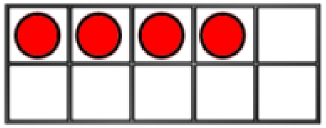


$4 + 3 = 7$

First | Then | Now







$7 - 3 = 4$

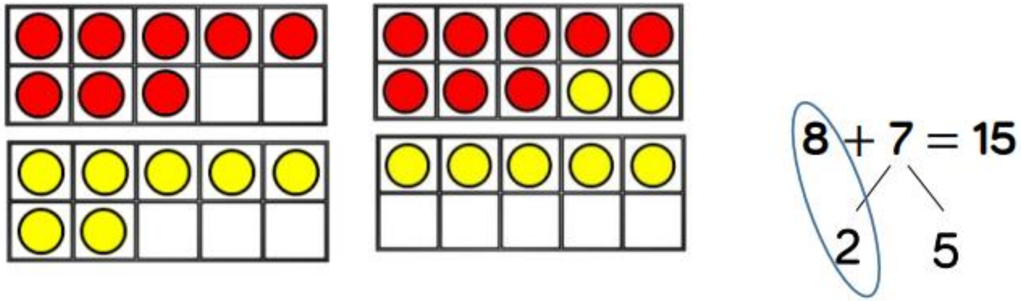
Benefits

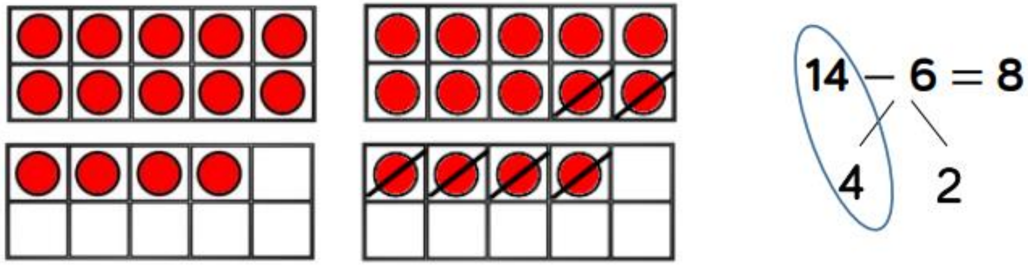
When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

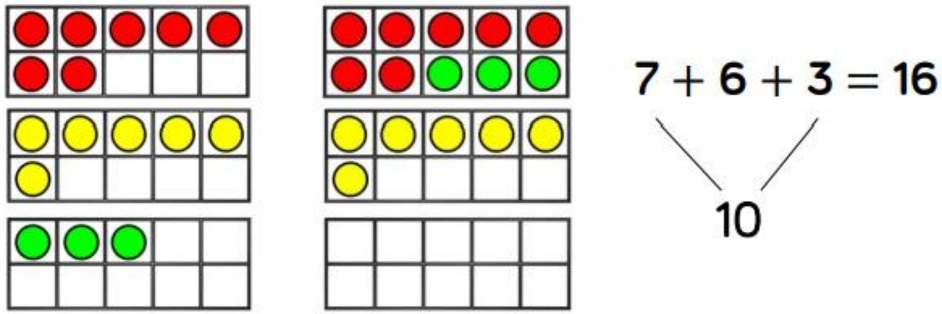
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts.
Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)







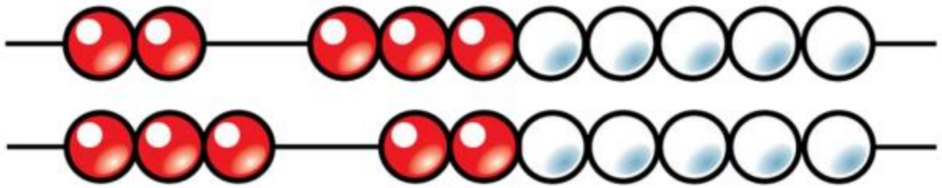
Benefits

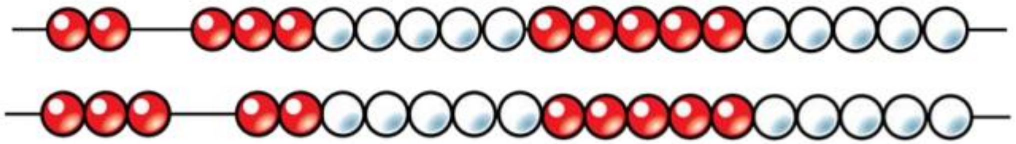
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

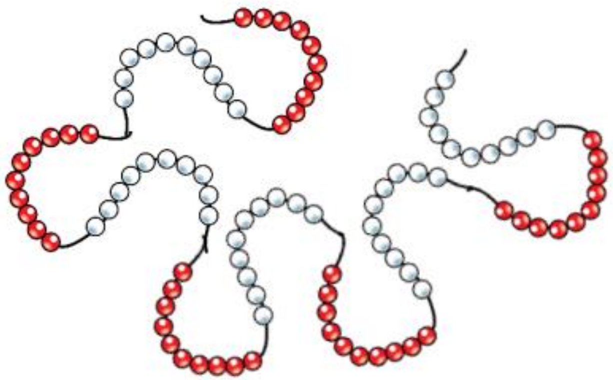
When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Bead Strings







Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

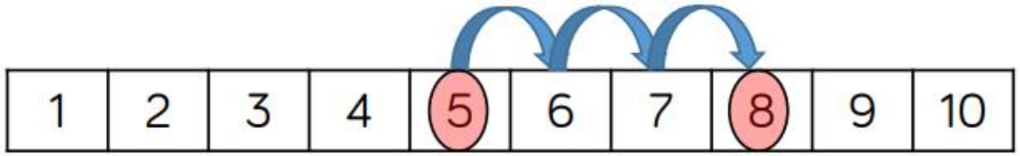
Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2 + 8 = 10$, move one bead, $3 + 7 = 10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

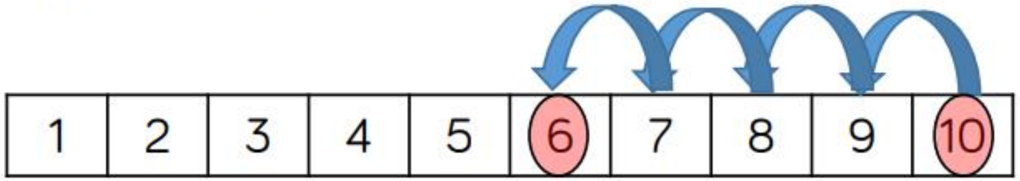
Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Number Tracks

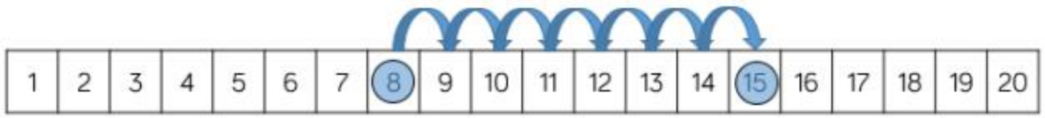
5 + 3 = 8



10 − 4 = 6



8 + 7 = 15



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

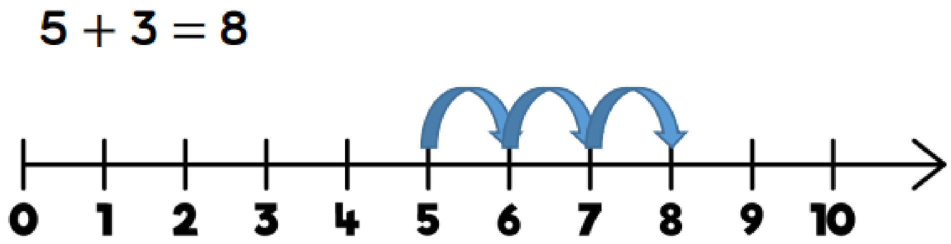
When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

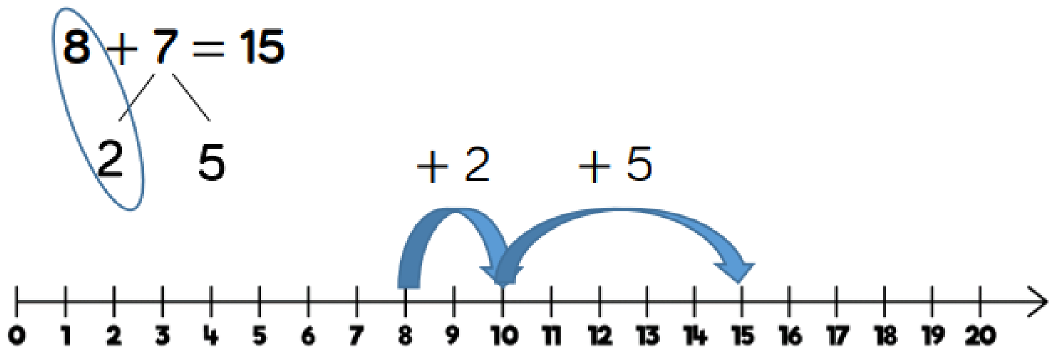
When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

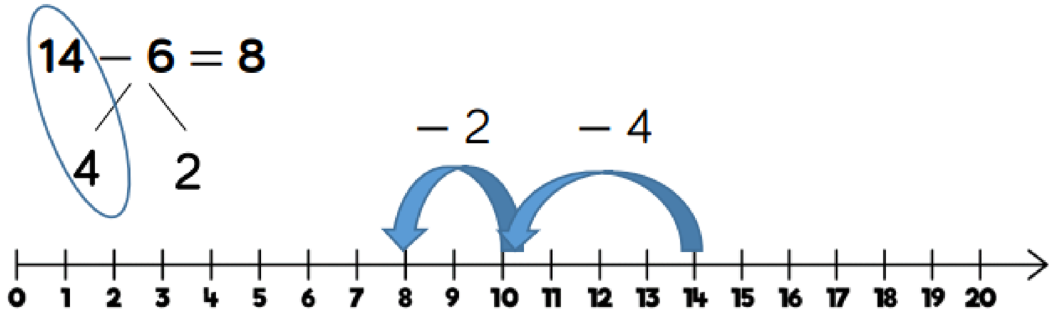
Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

Number Lines (labelled)







Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

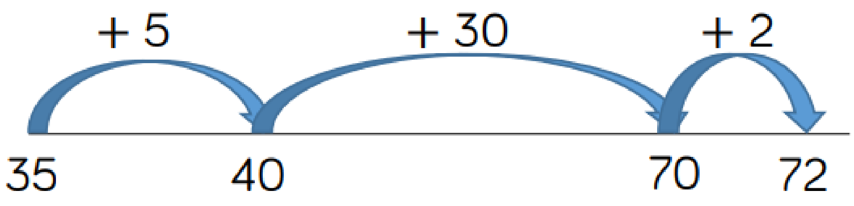
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

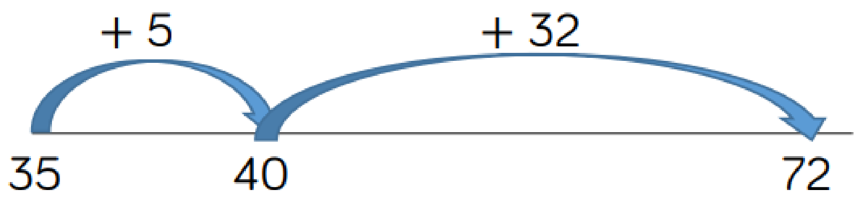
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

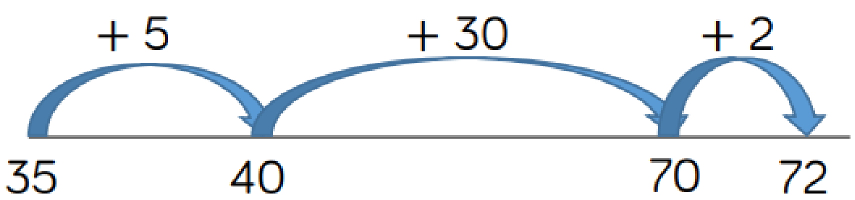
35 + 37 = 72



35 + 37 = 72



72 – 35 = 37



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

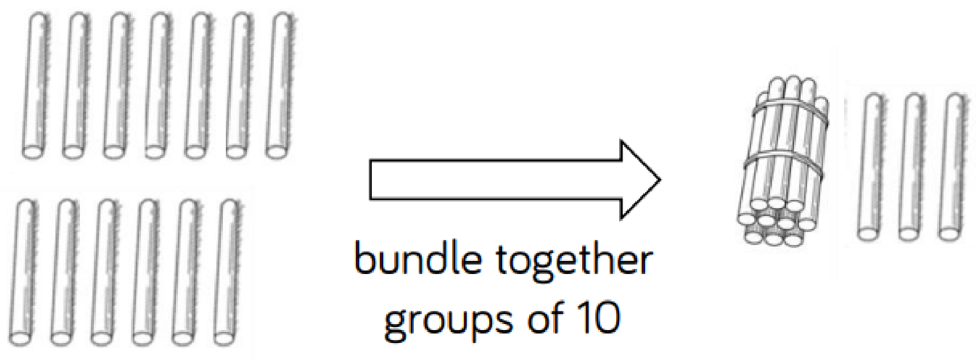
Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

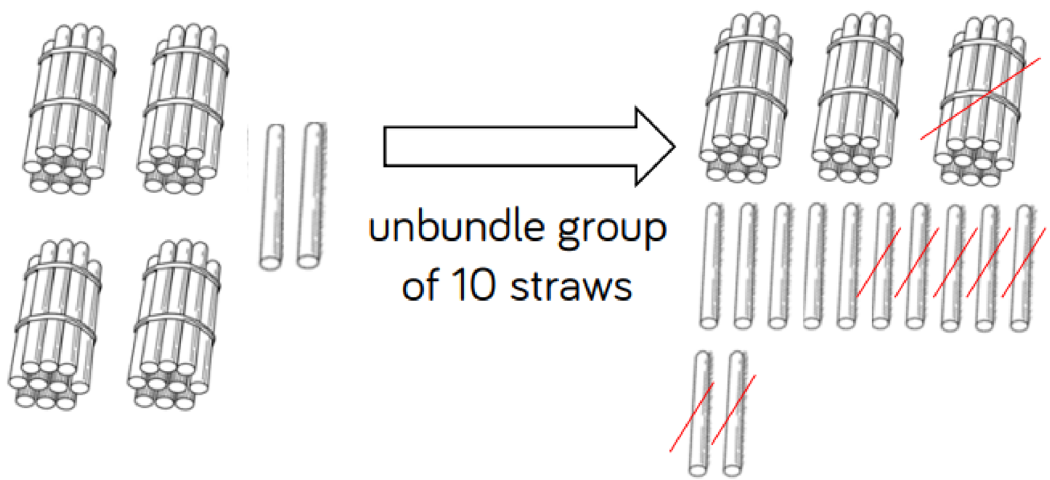
Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Straws

$7 + 6 = 13$



$42 - 17 = 25$



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

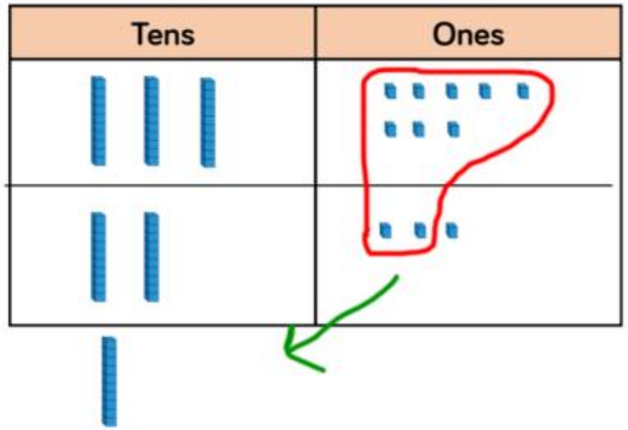
Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

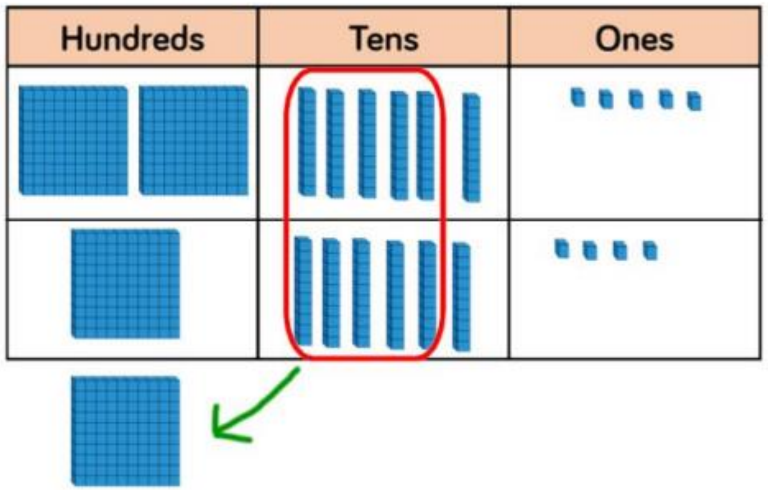
When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

Base 10/Dienes (addition)




```
  38
+ 23
----
  61
----
 1
```




```
  265
+ 164
-----
  429
-----
 1
```

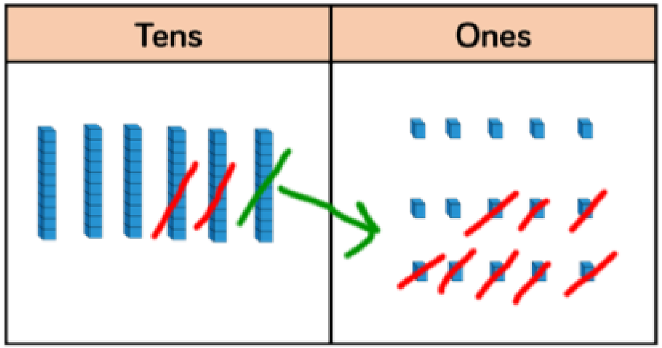
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

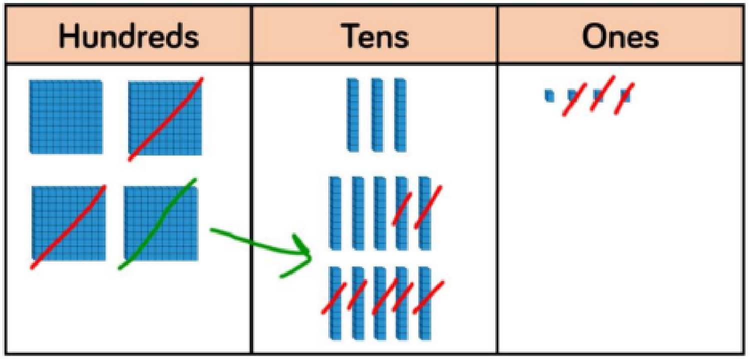
Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

Base 10/Dienes subtraction)




```
 5 1
  65
- 28
----
  37
```




```
 3 1
  435
- 273
-----
  262
```

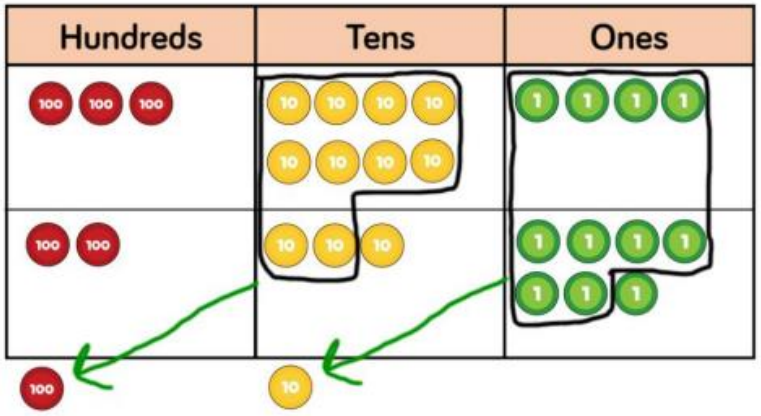
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

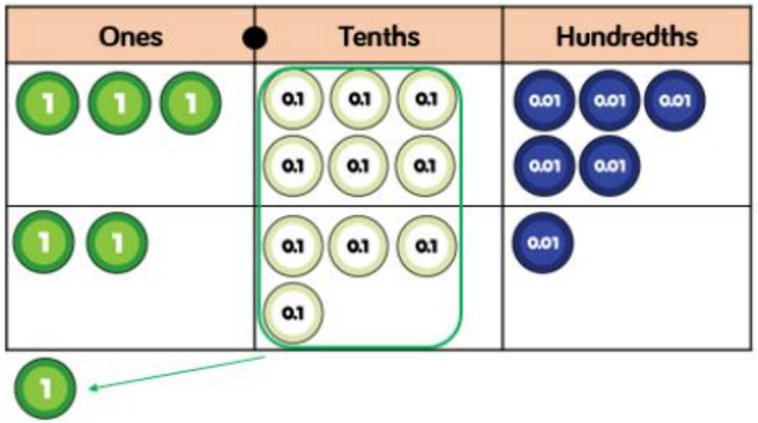
Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)



384
+ 237
621
1 1



3.65
+ 2.41
6.06
1

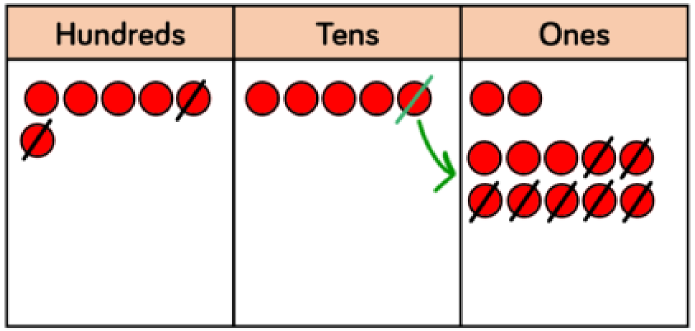
Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

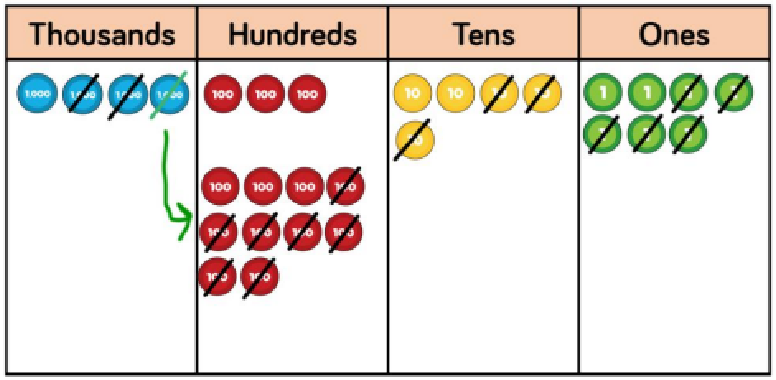
Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (subtraction)




```
   4 1
  652
– 207
  445
```




```
 3 1
  4357
– 2735
  1622
```

Benefits

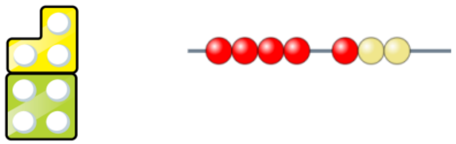
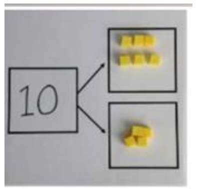
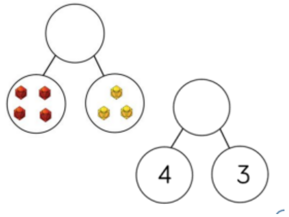
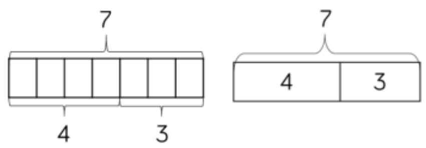
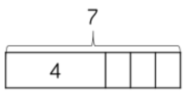
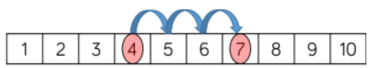
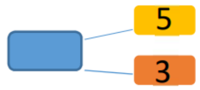
Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

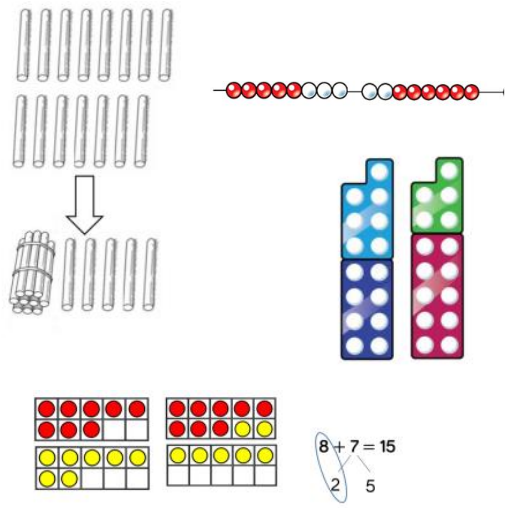
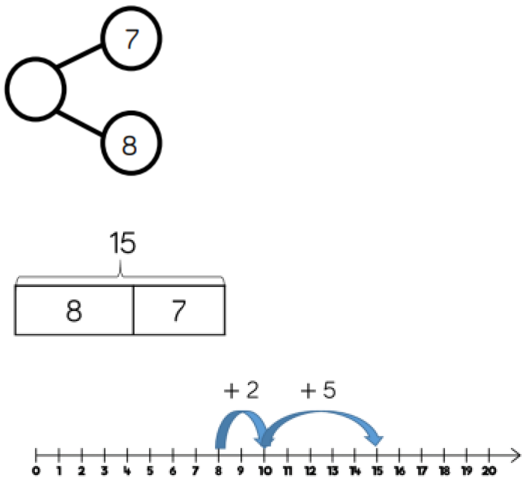
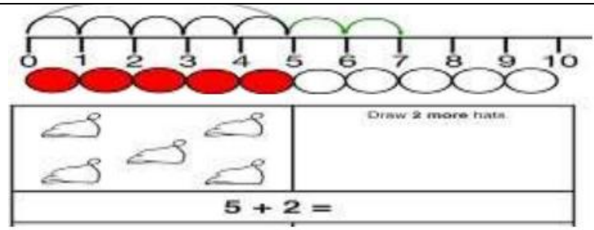
Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

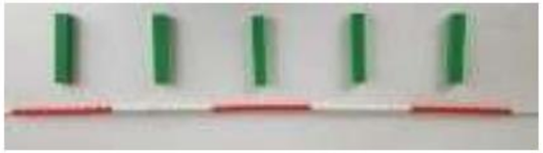
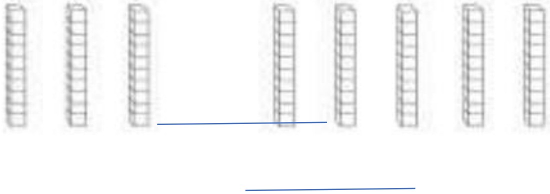
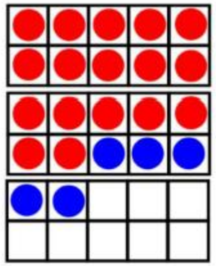
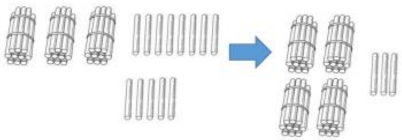
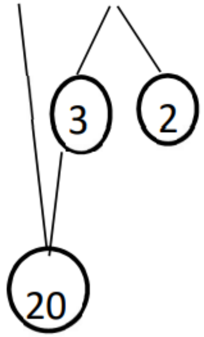
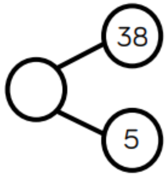
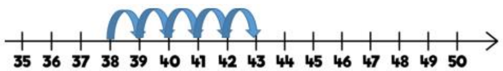
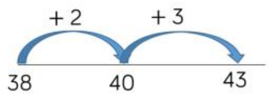
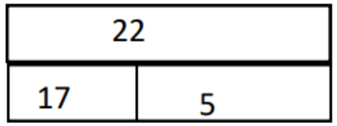
ADDITION

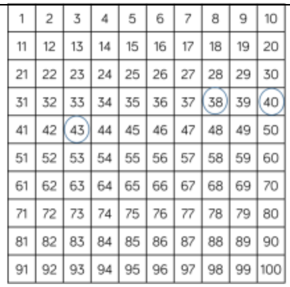
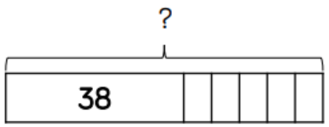
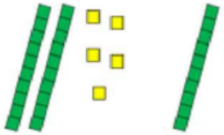
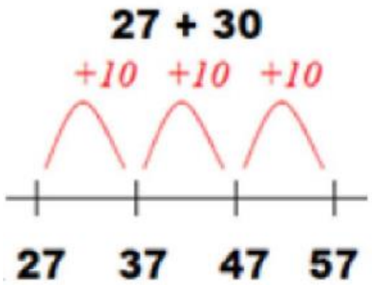
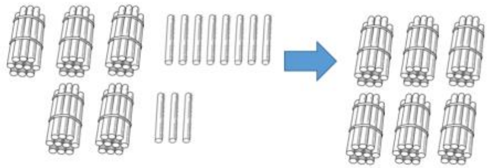
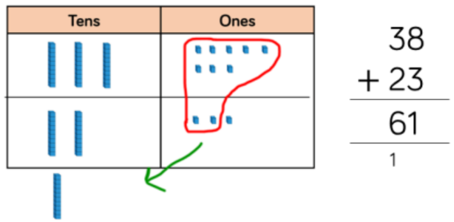
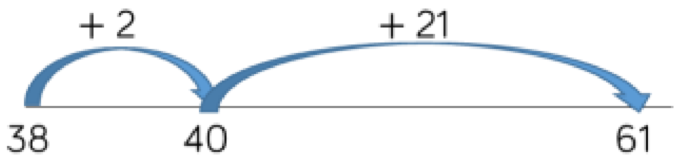
Addition

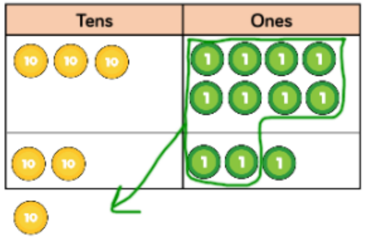
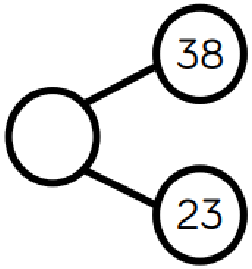
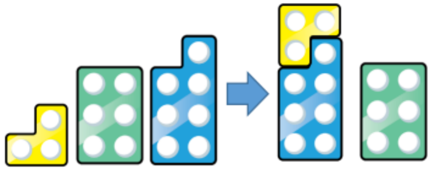
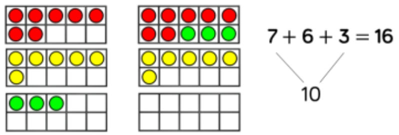
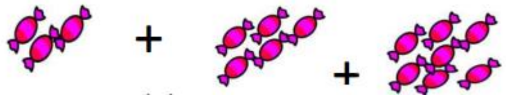
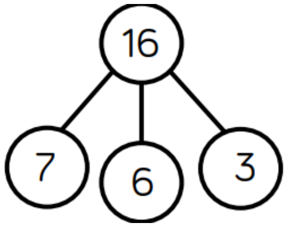
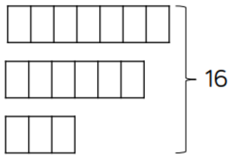
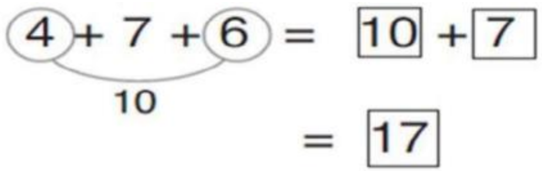
YEAR 1			
Objective	**Concrete**	**Pictorial**	**Abstract**
Add 1-digit numbers within 10	When adding numbers to 10, children can explore both aggregation and augmentation     Use part-part whole model Use cubes to add two numbers together as a group or in a bar 	Use pictures to add two numbers together as a group or bar The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation   The combination bar model, ten frame, bead string and number track all support augmentation  	$8 = 5 + 3$ $5 + 3 = 8$  Use the part-part whole diagram as shown to move into the abstract Include missing number questions to support varied fluency $8 = ? + 3$ $5 + ? = 8$

Add 1 and 2-digit numbers to 20			8 + 7 = 15
Represent and use number bonds and related subtraction facts within 20	 2 more than 5		Include missing number questions 8 = ? + 3 5 + ? = 8 Emphasise should be on the language '1 more than 5 is equal to 6' '2 more than 5 is 7' 8 is 3 more than 5'

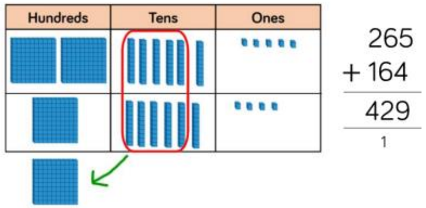
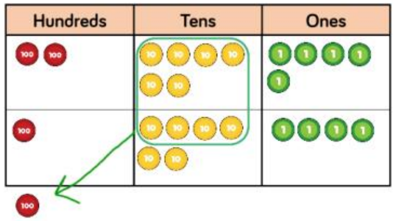
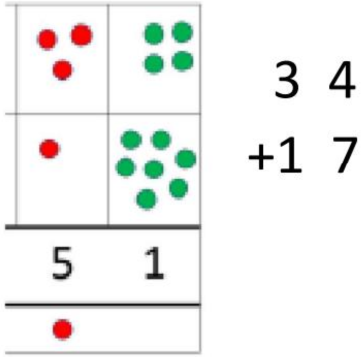
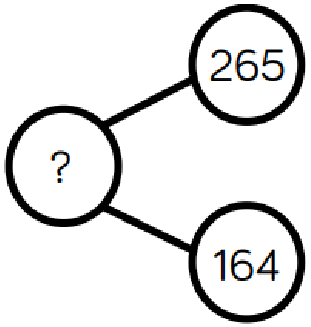
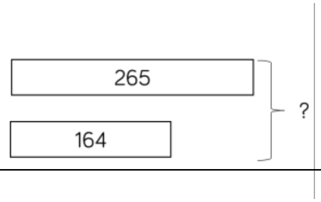
Addition

YEAR 2			
Objective	**Concrete**	**Pictorial**	**Abstract**
Adding multiples of ten	50 = 30 = 20  Model using dienes and bead strings	 3 tens + 5 tens = tens 30 + 60 = Use representations for base ten	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Add 1-digit and 2-digit numbers to 100	 17 + 5 = 22 Use ten frame to make 'magic' ten When adding single digits to a two-digit number, children should be encouraged to count on from the larger number. They should also apply their knowledge of number bonds to add more efficiently e.g. 8 + 5 = 13 so 38 + 5 = 43. 	17 + 5 = 22    	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22 – 17 = 5 22 – 5 = 17  Lead into recodring in column format, to reinforce place value and prepare children for formal written methods with larger values

			
Add a two-digit number and tens	 25 + 10 = 35 Explore that the ones digits does not change		27 + 10 = 37 27 + 20 = 47 27 + ☐ = 57
Add two two-digit numbers to 100	Model using dienes, place value counters and numicon  	Use number line and part-whole if necessary Children can use a blank number line and other representations to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient. 	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72 Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values

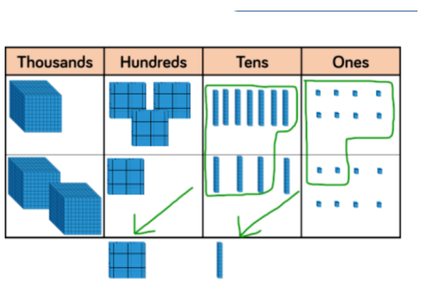
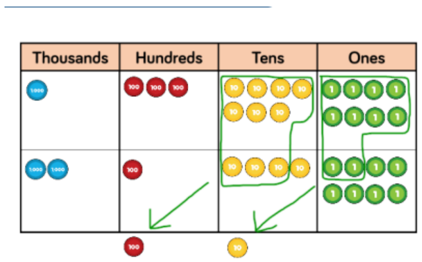
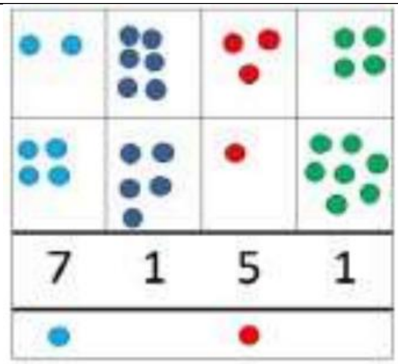
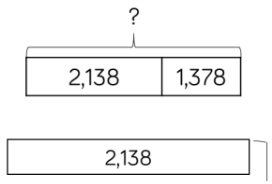
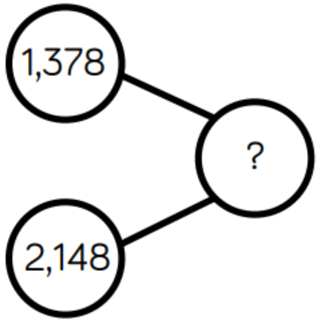
	 	 	
Add three one-digit numbers	Manipulatives that highlight number bonds to 10 are effective when adding three 1 -digit numbers  Combine to make 10 first if possible, or bridge 10 then add third digit. This supports children in their understanding of commutativity.   	 Regroup and draw representations    	 Combine the two numbers that make / bridge twn then add on the third 7 + 6 + 3 = 16

Addition

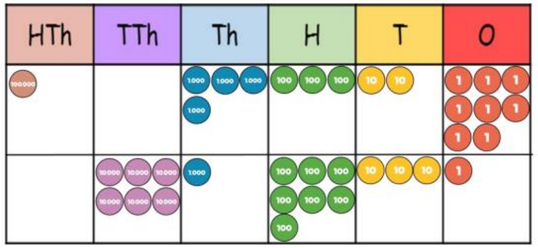
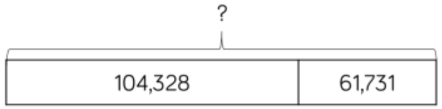
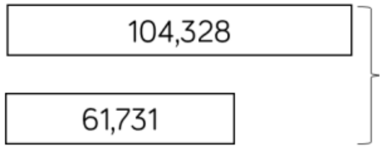
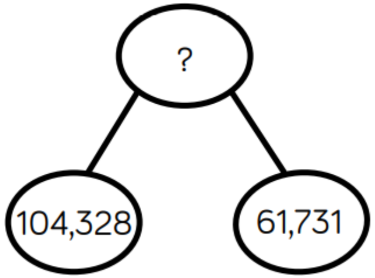
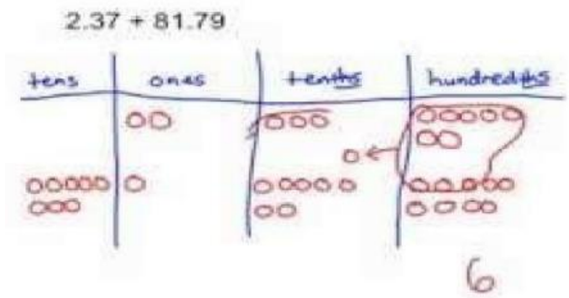
YEAR 3			
Objective	**Concrete**	**Pictorial**	**Abstract**
Add two or three two-digit or three-digit numbers	Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.   	 Children can draw representation of the grid to further support their understanding, carrying the ten **underneath** the line   	$20 + 5$ $40 + 8$ $60 + 13 = 73$ Start by partitioning the numbers before formal column to show the exchange 536 + 85 621 1 1 Add the ones first, then the tens, then the hundreds

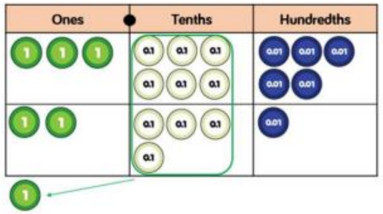
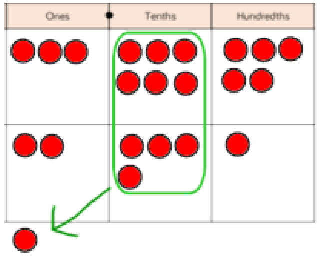
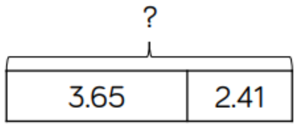
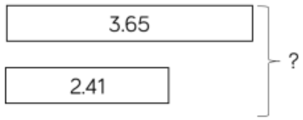
Estimate the answers to questions and use inverse operations to check answers	 Estimating 98 + 17 = ? 100 + 20 = 120	 Use number lines to illustrate estimation	Building up known number facts and using them to illustrate the inverse and to check answers: 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18

Addition

YEAR 4			
Objective	**Concrete**	**Pictorial**	**Abstract**
Add numbers with up to four digits	Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits  	 Draw representations using place value grid  	1 3 7 8 + 2 1 4 8 3 5 2 6 1 1

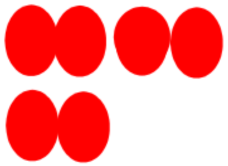
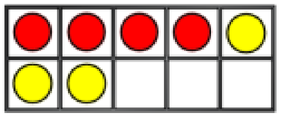
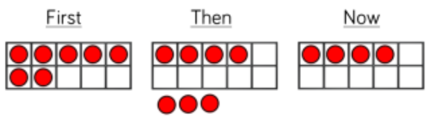
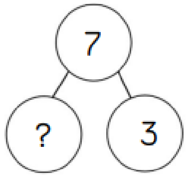
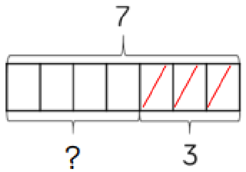
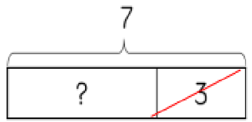
Addition

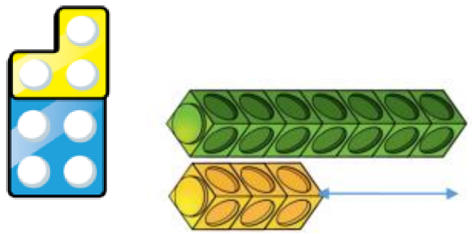
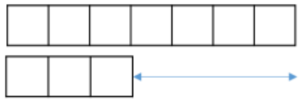
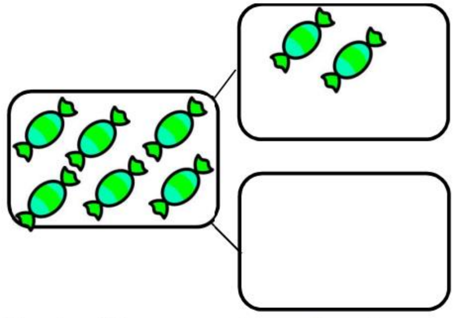
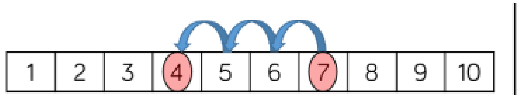
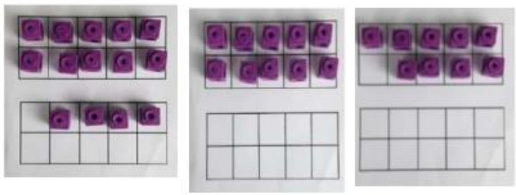
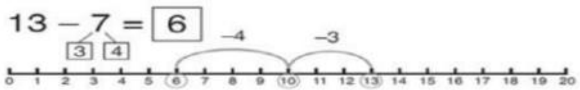
YEARS 5/6			
Objective	**Concrete**	**Pictorial**	**Abstract**
Add numbers with more than four digits	Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits 		At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently 1 0 4 3 2 8 + 6 1 7 3 1 1 6 6 0 5 9 1
Add with up to three decimal places	Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.		Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 3.65 + 2.41 6.06 1

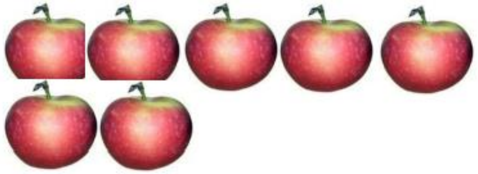
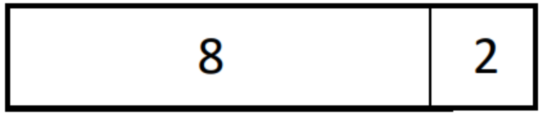
	Ones Tenths Hundredths Ones Tenths Hundredths	? 3.65 2.41 3.65 2.41 ?	

SUBTRACTION

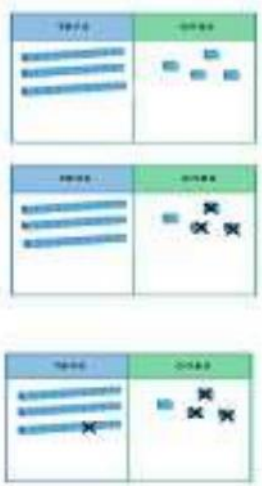
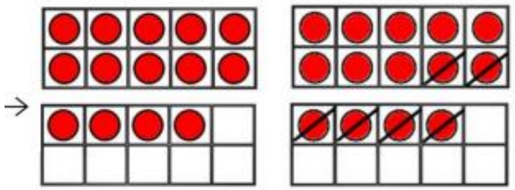
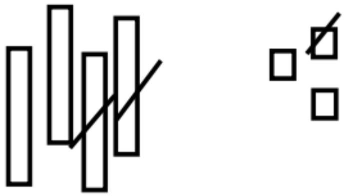
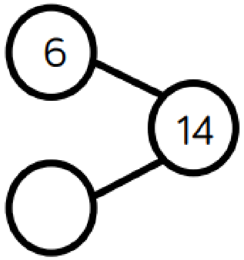
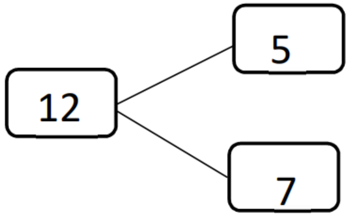
Subtraction

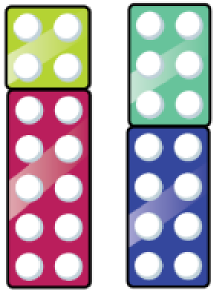
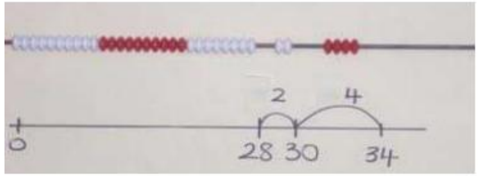
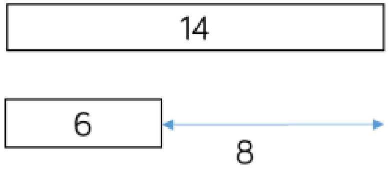
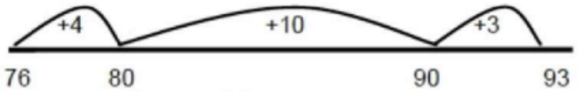
YEAR 1			
Objective	**Concrete**	**Pictorial**	**Abstract**
Subtract 1-digit numbers within 10	 $4 - 2 = 2$  $6 - 4 = 2$  	  	$7 - 4 = 3$

		   	
Make 10	14 – 9  Make 14 on the ten frame. Take 4 away to make ten, then take ten more away so that you have taken 5.	 13—7 Jump back 3 first, then another 4. Use ten as the stopping point.	16 – 8 How many do we take off first to get to 10? How many left to take off?

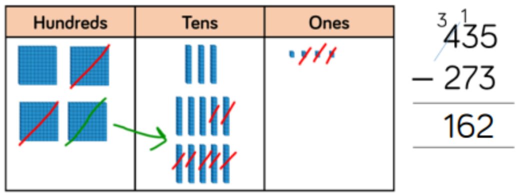
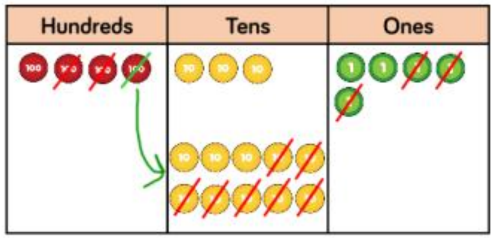
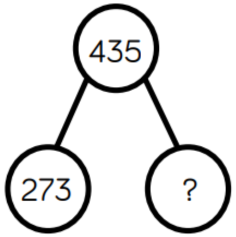
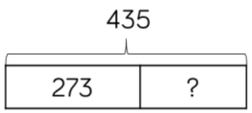
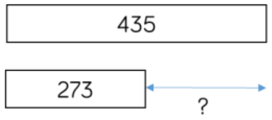
Bar model Including the inverse operations	 5 – 2 = 3		 10 = 8 + 2 10 = 2 + 8 10 – 2 = 8 10 – 8 = 2

Subtraction

YEAR 2			
Objective	**Concrete**	**Pictorial**	**Abstract**
Subtract 1 and 2-digit numbers	34 – 13 = 21  	Children draw representations of dienes and cross off  43 – 21 = 22  	43 – 21 = 22

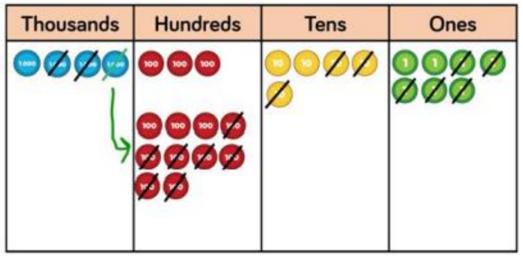
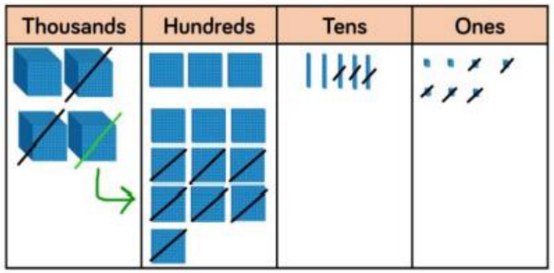
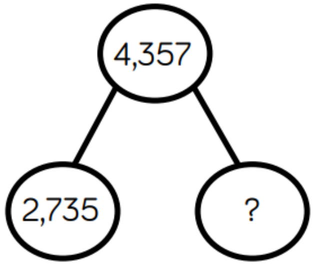
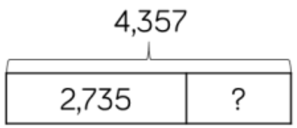
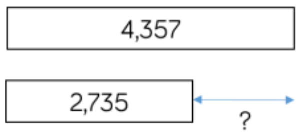
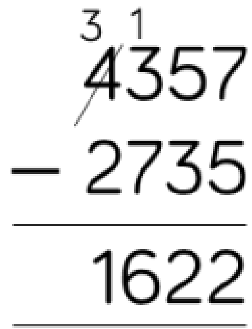
	  	  ‘counting on’ to find ‘difference’ Use the number line to count on to the next ten and then the rest	

Subtraction

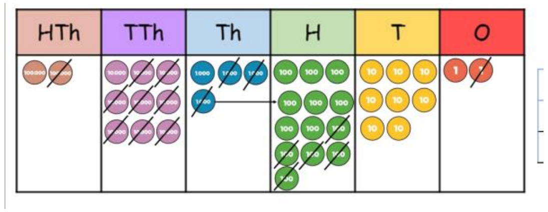
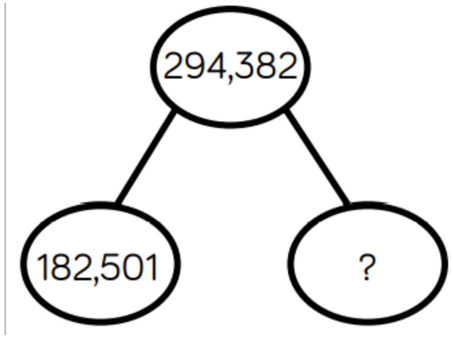
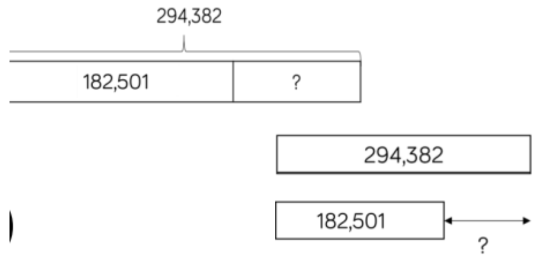
YEAR 3			
Objective	**Concrete**	**Pictorial**	**Abstract**
Subtract numbers with up to 3-digits	Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.  	  	Vary the position of the answer and question Expose children to missing number questions and vary the missing part of the calculation 678 = ? -1 688 – 10 = ? 678 = ? – 100

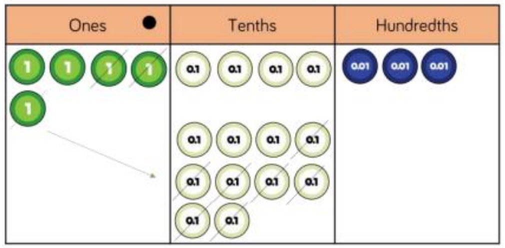
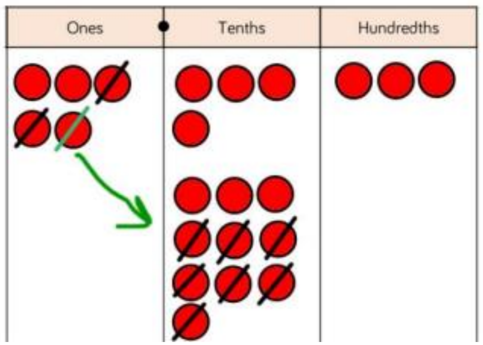
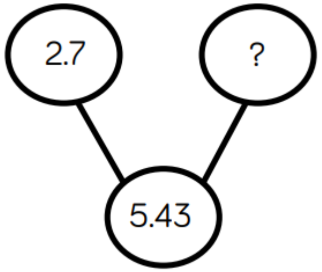
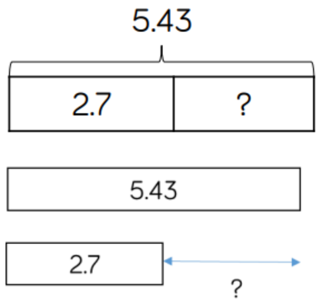
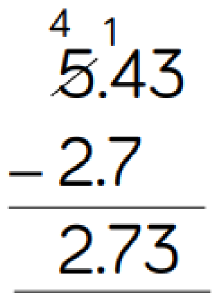
Subtraction

YEAR 4

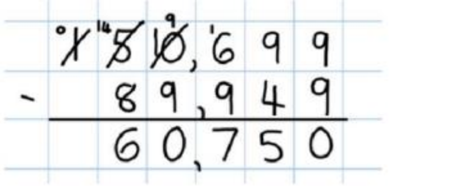
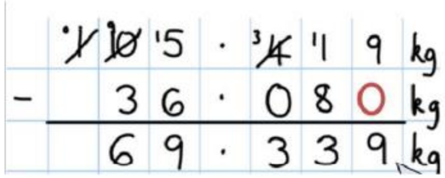
Objective	**Concrete**	**Pictorial**	**Abstract**
Subtract numbers with up to four digits *Introduce decimal subtraction through context of money*	Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.  	  	

Subtraction

<table>
<tr><th colspan="4"><u>YEAR 5</u></th></tr>
<tr><th><u>Objective</u></th><th><u>Concrete</u></th><th><u>Pictorial</u></th><th><u>Abstract</u></th></tr>
<tr>
<td>Subtract numbers with more than 4-digits</td>
<td>Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.
</td>
<td>
</td>
<td>At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently
<table>
<tr><td></td><td>2</td><td>9</td><td>3 ~~4~~</td><td>13</td><td>8</td><td>2</td></tr>
<tr><td>−</td><td>1</td><td>8</td><td>2</td><td>5</td><td>0</td><td>1</td></tr>
<tr><td></td><td>1</td><td>1</td><td>1</td><td>8</td><td>8</td><td>1</td></tr>
</table>
</td>
</tr>
</table>

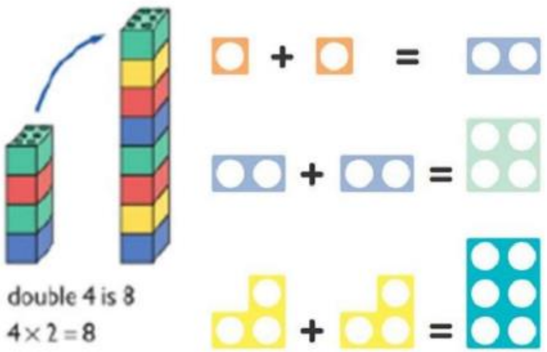
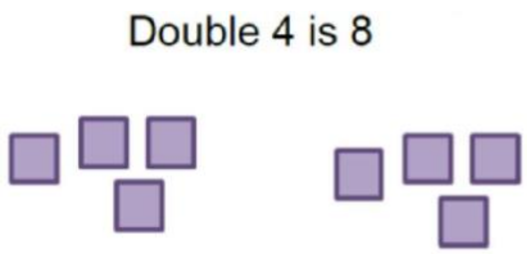
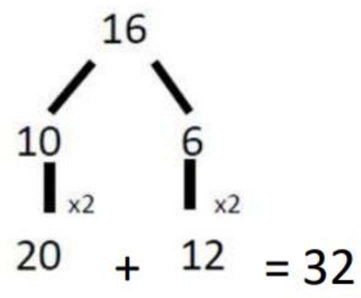
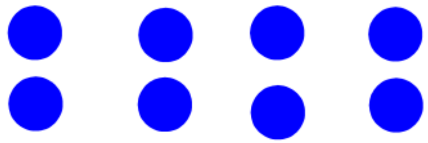
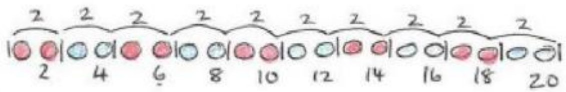
Subtract with up to three-decimal places	Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places  	 	

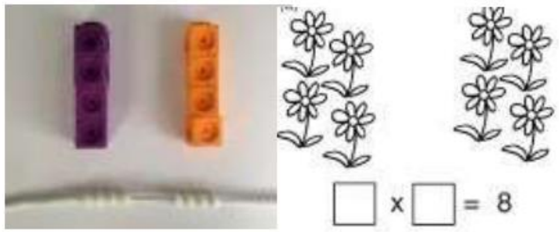
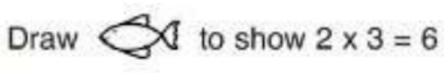
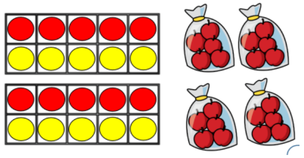
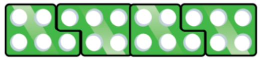
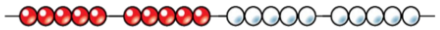
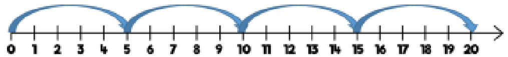
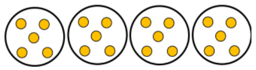
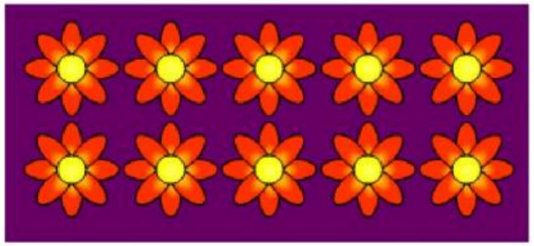
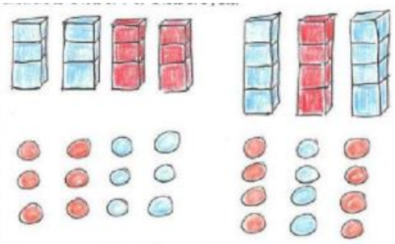
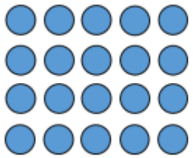
Subtraction

YEAR 6			
Objective	**Concrete**	**Pictorial**	**Abstract**
Subtract with increasingly large and more complex numbers and decimal values			 

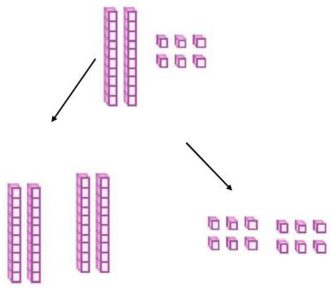
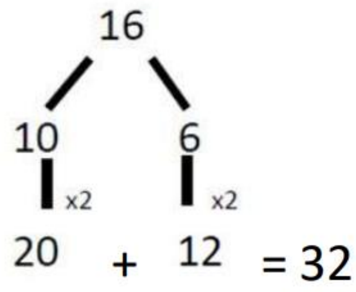
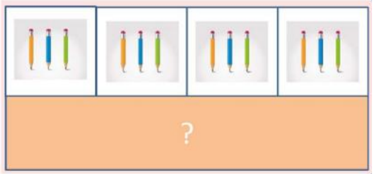
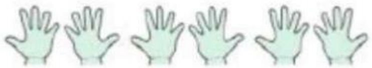
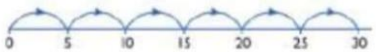
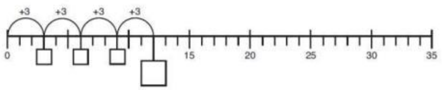
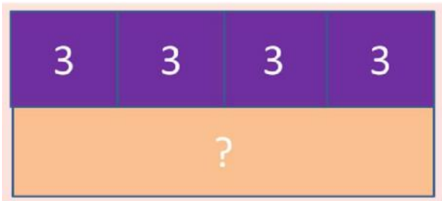
MULTIPLICATION

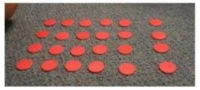
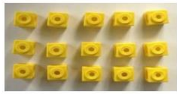
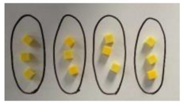
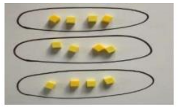
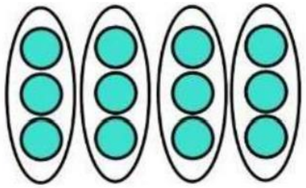
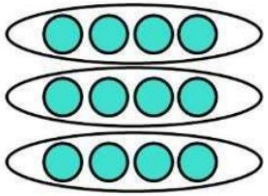
Multiplication

YEAR 1			
Objective	**Concrete**	**Pictorial**	**Abstract**
Doubling	Use practical activities using manipulatives, including cubes and number shapes to demonstrate doubling 	Draw pictures to show how to double numbers 	Partition a number and then double each part before recombining it back together 
Counting in multiples	Count the groups as children are skip counting, children may use their fingers ad they are skip counting  	 Children make representations to show counting in multiples 	Count in multiples of a number aloud. Write sequences with multiples of numbers 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting the total	Use manipulatives to create equal groups	Draw and make representations	2 x 4 = 8

			
Repeated addition	Use different objects to add equal groups   	Use pictorial including number lines to solve problems  	Write addition setences to describe objects and pictures 5 + 5 + 5 + 5 = 20
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2 etc 	Draw representations of arrays to show understanding  	3 x 2 = 6 2 x 5 = 10

Multiplication

YEAR 2			
Objective	**Concrete**	**Pictorial**	**Abstract**
Doubling	Model doubling using dienes and place value counters  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together 
Counting in multiples of 2, 3, 4, 5, 20 from 0	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples     	Count in multiples of a number aloud 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 Chikldren are introduced to the multiplication symbol $4 \times 3 = \square$

Multiplication is commutative	Create arrays using counters and cubes and number shapes    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer  	Use representations of arrays to show different calculations and explore commutativity  	12 = 3 x 4 12 = 4 x 3 One bag holds four apples. How many apples do 3 bags hold?

Multiplication

YEAR 3

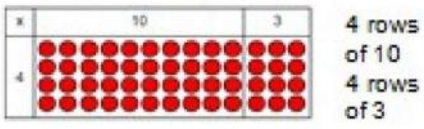
Objective

Grid method, progressing to the formal method

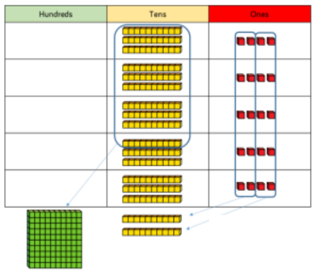
Multiply two-digit numbers by one-digit numbers

Concrete

Show the links with arrays to first introduce the grid method



Move onto base ten to move towards a more compact method



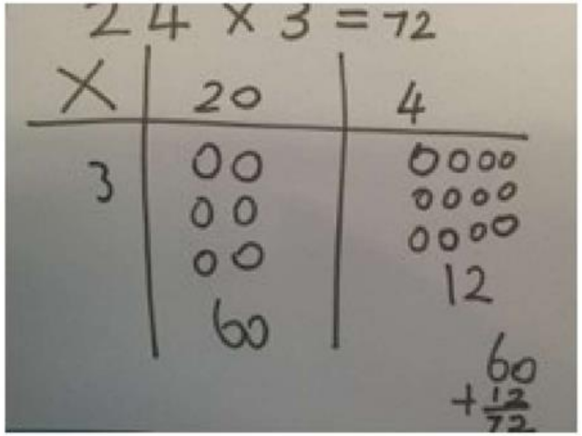
Move onto place value counters to show how we are finding groups of a number.

Place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge

Pictorial

Children can represent their work with place value counters in a way that they understand

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking



Bar model are used to explore missing numbers

Abstract

Start with multiplying by one-digit numbers and showing the clear addition alongside the grid

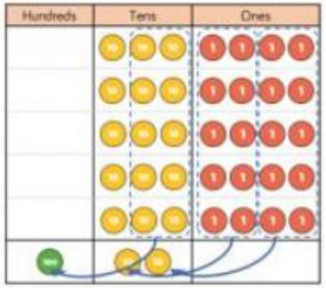
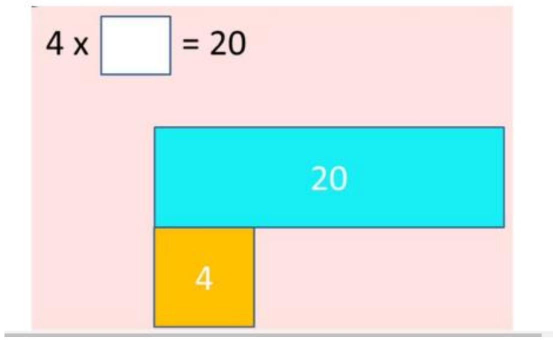
×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a two-digit number showing the different rows within the grid method

Expanded method is used

	H	T	O	
		3	4	
×			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

	Hundreds Tens Ones	4 x ☐ = 20 20 4	

Multiplication

YEAR 4

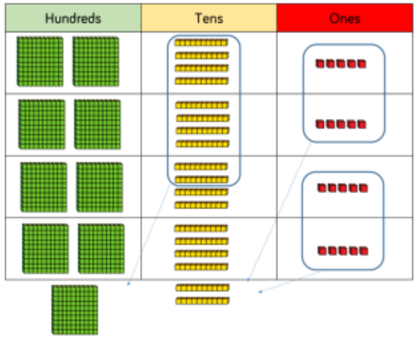
Objective	Concrete	Pictorial	Abstract

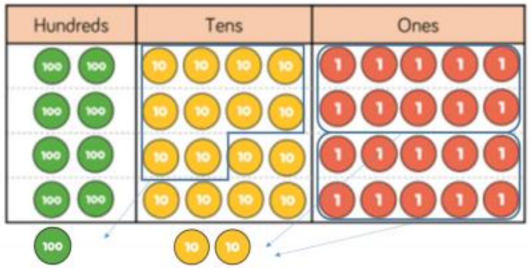
Objective

Grid method recap from Year 3 for two-digits multiplied by one-digit

Move to multiplying three-digit numbers by one-digit

Concrete

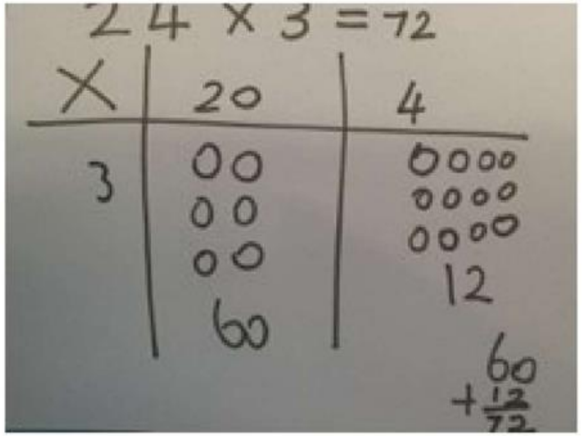




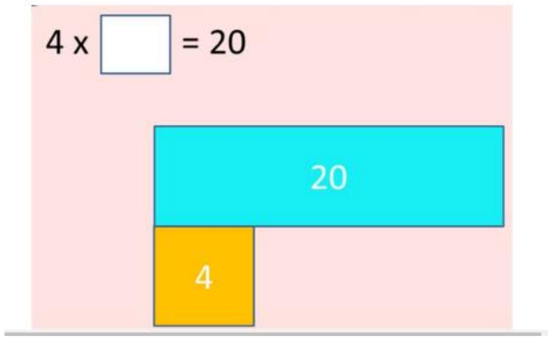
Pictorial

Children can represent their work with place value counters in a way that they understand

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking



Bar model are used to explore missing numbers



Abstract

Start with multiplying by one-digit numbers and showing the clear addition alongside the grid

×	30	5
7	210	35

210 + 35 = 245

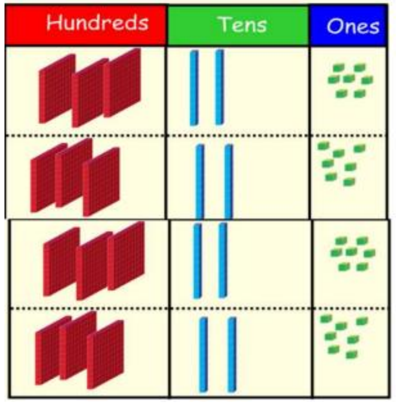
Short multiplication method

245 x 4 = 980

	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

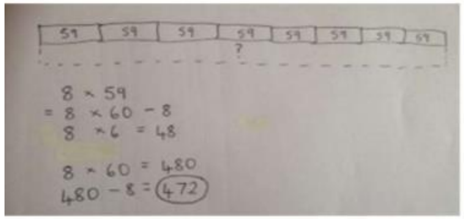
<table>
<tr>
<td>Column multiplication</td>
<td>Children can continue to be supported by place value counters at the stage of multiplication. This initially is done with there is no regrouping.

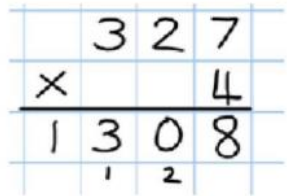
321 x 2 = 642



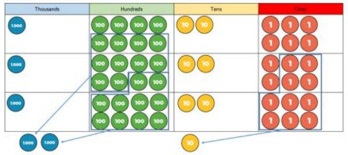
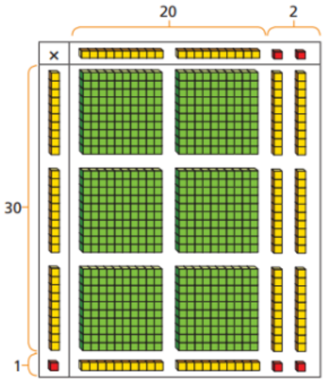
It is important at this stage that they always multiply the ones first

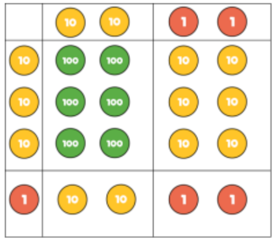
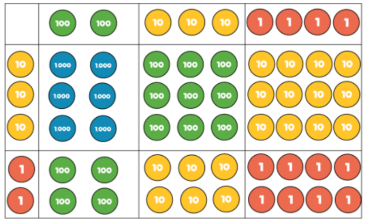
The corresponding long multiplication is modelled alongside</td>
<td><table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>
The grid method may be used to show how this relates to a formal written method



Bar modelling and number lines can support learners with solving problems with multiplication alongside the formal written methods</td>
<td>
</td>
</tr>
</table>

Multiplication

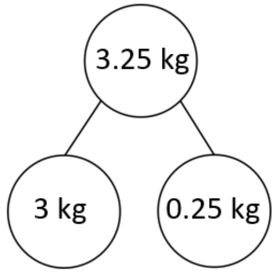
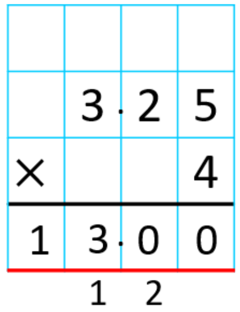
<table>
<tr><th colspan="4">YEAR 5</th></tr>
<tr><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Column multiplication for three and four-digits multiplied by one-digit</td>
<td>Manipulatives may still be used with the corresponding long multiplication modelled alongside
</td>
<td></td>
<td>1826 x 3 = 5478
<table>
<tr><td></td><td>Th</td><td>H</td><td>T</td><td>O</td></tr>
<tr><td></td><td>1</td><td>8</td><td>2</td><td>6</td></tr>
<tr><td>×</td><td></td><td></td><td></td><td>3</td></tr>
<tr><td></td><td>5</td><td>4</td><td>7</td><td>8</td></tr>
<tr><td></td><td>2</td><td></td><td>1</td><td></td></tr>
</table>
</td>
</tr>
<tr>
<td>Column multiplication for 2-digit multiplied by 2-digit</td>
<td>When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10.

</td>
<td>The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.
<table>
<tr><td>×</td><td>20</td><td>2</td></tr>
<tr><td>30</td><td>600</td><td>60</td></tr>
<tr><td>1</td><td>20</td><td>2</td></tr>
</table>
</td>
<td>22 x 31 = 682
<table>
<tr><td></td><td></td><td>2</td><td>2</td></tr>
<tr><td>×</td><td></td><td>3</td><td>1</td></tr>
<tr><td></td><td></td><td>2</td><td>2</td></tr>
<tr><td></td><td>6</td><td>6</td><td>0</td></tr>
<tr><td></td><td>6</td><td>8</td><td>2</td></tr>
</table>
</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column multiplication for 3-digit multiplied by 2-digit</td>
<td>Children can continue to use the area model when multiplying 3- digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.
</td>
<td>
<table>
<tr><td>×</td><td>200</td><td>30</td><td>4</td></tr>
<tr><td>30</td><td>6,000</td><td>900</td><td>120</td></tr>
<tr><td>2</td><td>400</td><td>60</td><td>8</td></tr>
</table>
</td>
<td>Children should now move towards the formal written method, seeing the links with the grid method.

234 x 32 = 7488
<table>
<tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr>
<tr><td></td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>×</td><td></td><td>3</td><td>2</td></tr>
<tr><td></td><td>4</td><td>6</td><td>8</td></tr>
<tr><td>$_{1}7$</td><td>$_{1}0$</td><td>2</td><td>0</td></tr>
<tr><td>7</td><td>4</td><td>8</td><td>8</td></tr>
</table>
</td>
</tr>
<tr>
<td>Column multiplication for 4-digit multiplied by 2-digit</td>
<td></td>
<td></td>
<td>If children are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

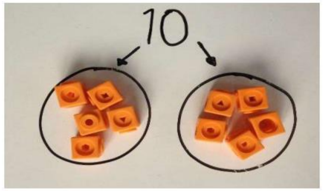
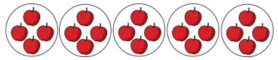
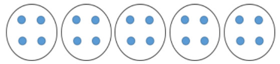
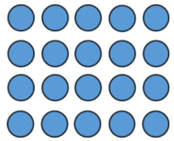
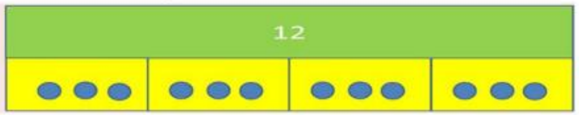
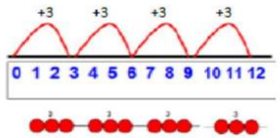
2739 x 28 = 76,692
<table>
<tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr>
<tr><td></td><td>2</td><td>7</td><td>3</td><td>9</td></tr>
<tr><td>×</td><td></td><td></td><td>2</td><td>8</td></tr>
<tr><td>$_{2}2$</td><td>$_{5}1$</td><td>$_{3}9$</td><td>$_{7}1$</td><td>2</td></tr>
<tr><td>$_{1}5$</td><td>4</td><td>$_{1}7$</td><td>8</td><td>0</td></tr>
<tr><td>7</td><td>6</td><td>6</td><td>9</td><td>2</td></tr>
<tr><td></td><td></td><td>1</td><td></td><td></td></tr>
</table>
Consider where exchanged digits are placed and make sure this is consistent.</td>
</tr>
</table>

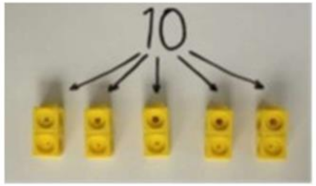
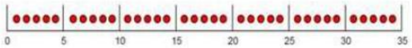
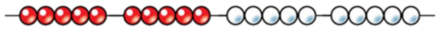
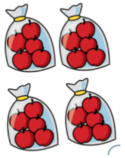
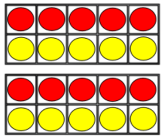
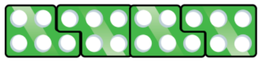
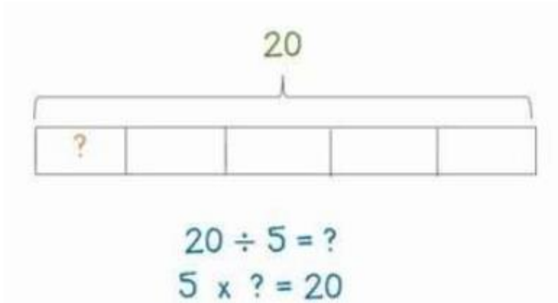
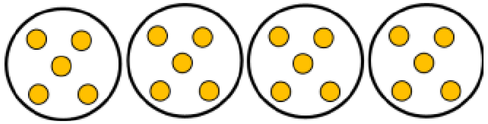
Multiplication

YEAR 6			
Objective	**Concrete**	**Pictorial**	**Abstract**
Multiplying decimals up to two-decimal places by a single digit		A jar of sweets has a mass of 3.25 kg. What is the total mass of 4 jars of sweets?  $3 \times 4 + 0.25 \times 4$ $12 + 1$	Remind children that the single digit belongs in the ones column. 

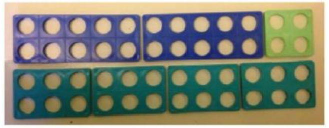
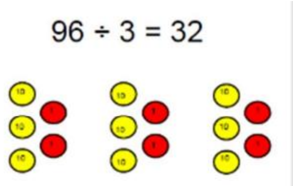
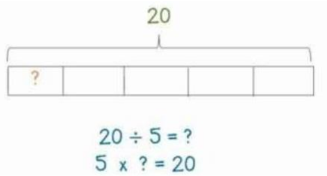
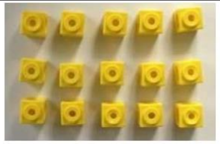
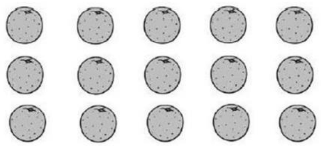
DIVISION

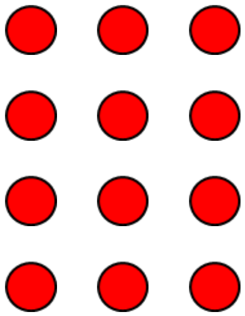
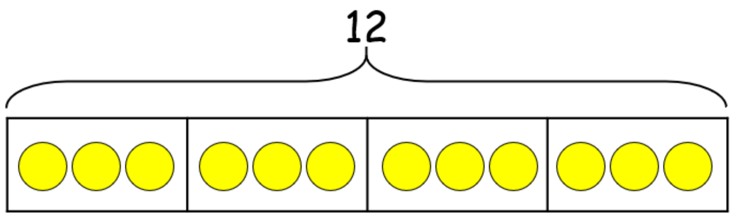
Division

YEAR 1			
Objective	**Concrete**	**Pictorial**	**Abstract**
Division as sharing	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities  8 shared between 2 is 4  20 shared between 5 is 4   Children use bar modelling to show and support understanding  	12 shared between 3 is 4 There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?
Division as grouping	Children solve problems by grouping and counting the number of groups. Children can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.	Grouping encourages children to count in multiples and links to repeated subtraction on a number line.  	Divide 28 into 7 groups. How many are in each group? There are 20 apples altogether. They are put in bags of 5. How many bags are there?

	     	Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group  	

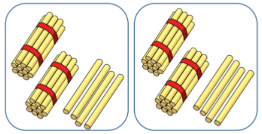
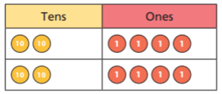
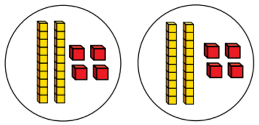
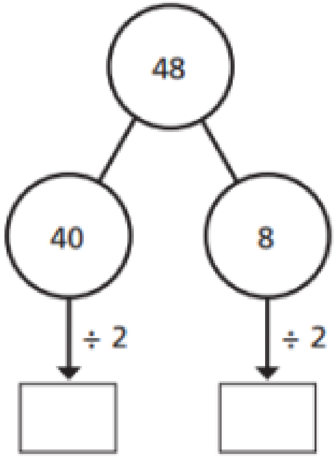
Division

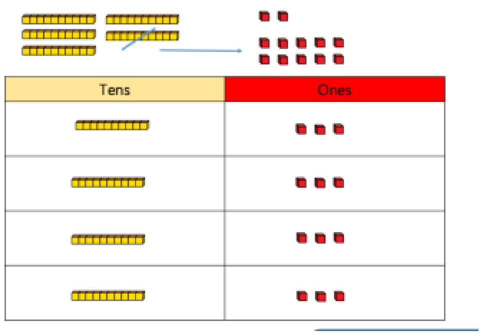
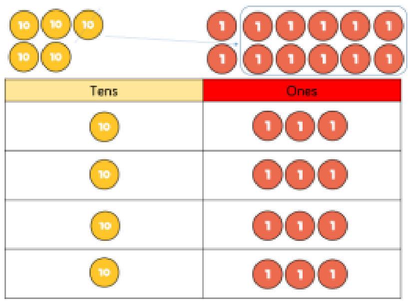
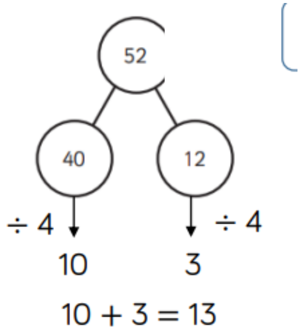
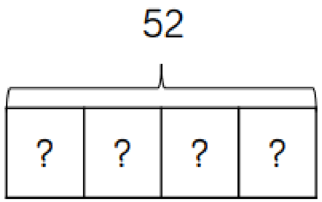
YEAR 2			
Objective	**Concrete**	**Pictorial**	**Abstract**
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding  24 divided into groups of 6 = 4 	Continue to use bar modelling to aid solving division problems 	How many groups of 6 in 24? Children are introduced to the division symbol 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinkking about the number setences that can eb created e.g. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7

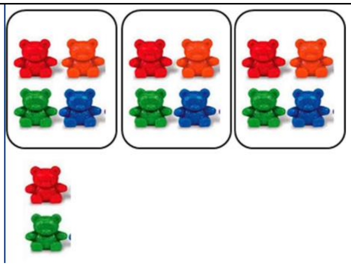
			4 = 28 ÷ 7 7 = 28 ÷ 4
Division as sharing equally	12 counters shared equally between 4  There are 12 counters altogether. I have shared them into 4 equal groups. There are 3 in each group	Rosie has 12 balloons. She shares them between 4 bags. How many balloons are in each bag? 	Complete the division 12 ÷ □ = □

Division

YEAR 3

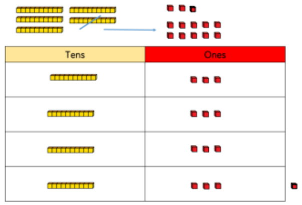
Objective	Concrete	Pictorial	Abstract
Divide 2-digits by 1-digit (sharing with no exchange)	When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones. Straws, Base 10 and place value counters can all be used to share numbers into equal groups.   	Part-whole models can provide children with a clear written method that matches the concrete representation. 	48 ÷ 2 = 24

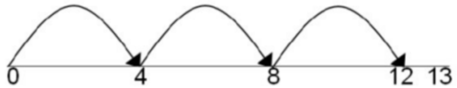
Divide 2-digits by 1-digit (sharing with exchange)	When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.  	 Flexible partitioning in a part-whole model supports this method. 	52 ÷ 4 = 13
Division with remainders	14 ÷ 3 = Divide objects between groups and see how much is left over	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder	Complete written divisions and show the remainder using r



When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones.

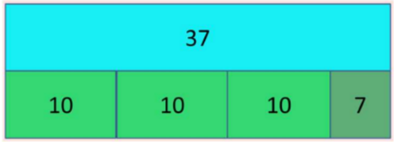
Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made.

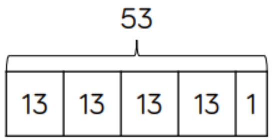




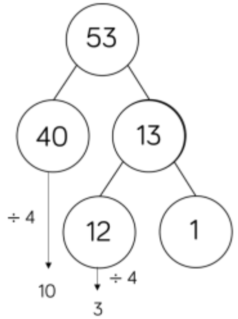
Draw dots and group them to divide an amount and clearly show a remainder

Use bar models to show division with remainders



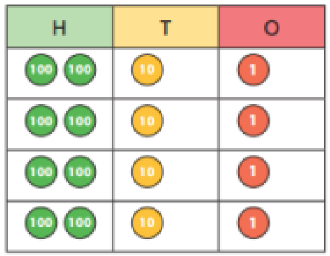
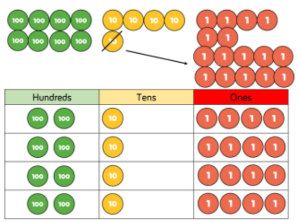
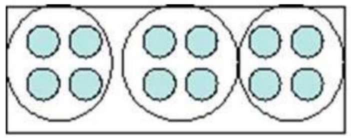
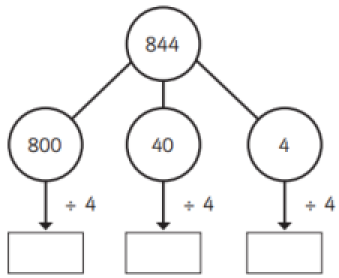


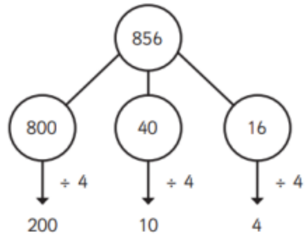
Flexible partitioning in a part-whole model supports this method.



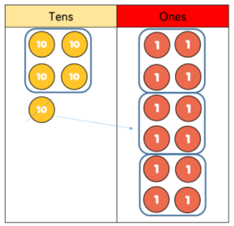
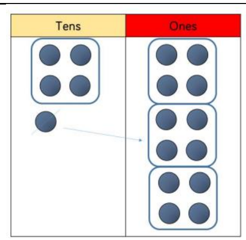
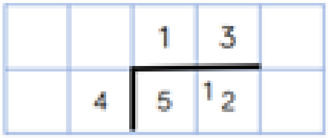
53 ÷ 4 = 13 r1

Division

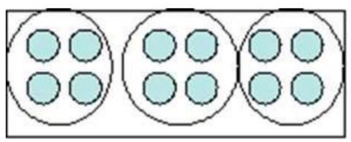
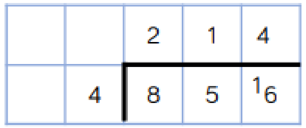
YEAR 4			
Objective	**Concrete**	**Pictorial**	**Abstract**
Divide at least three-digit numbers by one-digit (sharing)	Children can continue to use place value counters to share 3- digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. 844 ÷ 4 = 211  856 ÷ 4 = 214 	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups  Flexible partitioning in a part-whole model supports this method 	844 ÷ 4 = 211

		856 800 40 16 ÷ 4 ÷ 4 ÷ 4 200 10 4	

Division

<table>
<tr><th colspan="4">YEAR 5</th></tr>
<tr><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Divide 2-digits by 1-digit (grouping)</td>
<td>When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?
Remainders can also be seen as they are left ungrouped.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Divide three-digit numbers by one-digit (grouping)

Short Division</td>
<td>Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters can be used on a place value grid to support this understanding.</td>
<td>Children can also draw their own counters and group them through a more pictorial method
</td>
<td>856 ÷ 4 = 214
</td>
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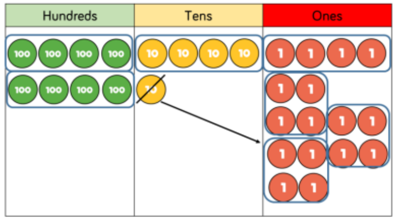
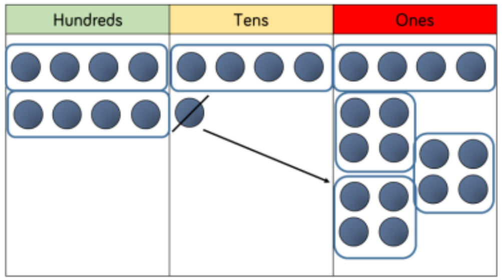
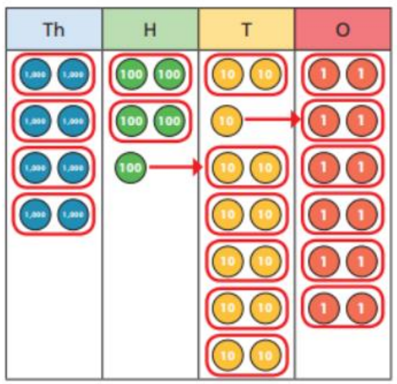
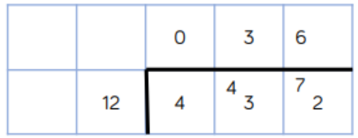
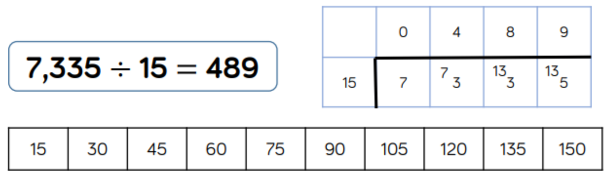
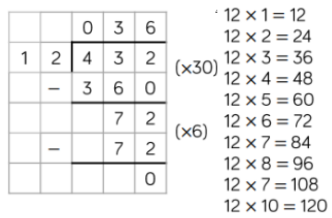
	  	Encourage them to move towards counting in multiples to divide more efficiently	
Divide 4-digits by 1-digit (grouping)	Place value counters or plain counters can be used on a place value grid to support children to divide 4- digits by 1-digit.  	Children can also draw their own counters and group them through a more pictorial method.	8532 ÷ 2 = 4266 4 2 6 6 2 | 8 5 $^{1}3$ $^{1}2$ Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

Division

<table>
<tr><th colspan="4"><u>YEAR 6</u></th></tr>
<tr><th><u>Objective</u></th><th><u>Concrete</u></th><th><u>Pictorial</u></th><th><u>Abstract</u></th></tr>
<tr><td>Divide multi-digits by -digits (short division)</td><td colspan="2">When children begin to divide up to 4- digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective.</td><td>432 ÷ 12 = 36


Children can write out multiples to support their calculations with larger remainders.

</td></tr>
<tr><td>Divide multi-digits by -digits (long division)</td><td colspan="2"></td><td>Children can also divide by 2-digit numbers using long division. Children can write out multiples to support their calculations with larger remainders.

432 ÷ 12 = 36

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<table>
<tr>
<td></td>
<td></td>
<td>
7,355 ÷ 15 = 489
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<tr><td></td><td>0</td><td>4</td><td>8</td><td>9</td><td></td></tr>
<tr><td>15</td><td>7</td><td>3</td><td>3</td><td>5</td><td></td></tr>
<tr><td>–</td><td>6</td><td>0</td><td>0</td><td>0</td><td>(×400</td></tr>
<tr><td></td><td>1</td><td>3</td><td>3</td><td>5</td><td></td></tr>
<tr><td>–</td><td>1</td><td>2</td><td>0</td><td>0</td><td>(×80)</td></tr>
<tr><td></td><td></td><td>1</td><td>3</td><td>5</td><td></td></tr>
<tr><td>–</td><td></td><td>1</td><td>3</td><td>5</td><td>(×9)</td></tr>
<tr><td></td><td></td><td></td><td></td><td>0</td><td></td></tr>
</table>
1 × 15 = 15
2 × 15 = 30
3 × 15 = 45
4 × 15 = 60
5 × 15 = 75
10 × 15 = 150
</td>
</tr>
<tr>
<td>Divide multi-digits by 2-digits (long division)</td>
<td></td>
<td>
When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question.

372 ÷ 15 = 24 r12
<table>
<tr><td></td><td></td><td></td><td>2</td><td>4</td><td>r</td><td>1</td><td>2</td></tr>
<tr><td>1</td><td>5</td><td>3</td><td>7</td><td>2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>–</td><td>3</td><td>0</td><td>0</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>7</td><td>2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>–</td><td></td><td>6</td><td>0</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>1</td><td>2</td><td></td><td></td><td></td></tr>
</table>
1 × 15 = 15
2 × 15 = 30
3 × 15 = 45
4 × 15 = 60
5 × 15 = 75
10 × 15 = 150

372 ÷ 15 = 24 4/5
<table>
<tr><td></td><td></td><td></td><td>2</td><td>4</td><td>$\frac{4}{5}$</td></tr>
<tr><td>1</td><td>5</td><td>3</td><td>7</td><td>2</td><td></td></tr>
<tr><td></td><td>–</td><td>3</td><td>0</td><td>0</td><td></td></tr>
<tr><td></td><td></td><td></td><td>7</td><td>2</td><td></td></tr>
<tr><td></td><td>–</td><td></td><td>6</td><td>0</td><td></td></tr>
<tr><td></td><td></td><td></td><td>1</td><td>2</td><td></td></tr>
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